UCD School of Electrical, Electronic



& Communications Engineering

EEEN40010 Control Systems

Signed: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

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Minor Project 2 Control Systems Report

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Digital Control

## Q1 Approximate model

The fan and plat controller problem, where the input is the DC fan voltage and the output is is the angle which the plate makes with the downward vertical, is approximated by the following model:

I want to determine if is a valid operating point for the model, given the model parameters in lab manual. For

These parameters are given to validate the operating point:

Each equation is then given to validate the chosen operating point. The derivatives are zero and the offsets can be ignored.

Substituting model parameters, constants and values of the operating point into Eq(1)

Eq(1)

Eq(2)

Eq(3)

Eq(4)

Eq(5)

We note that the above equations are satisfied at the operating point and so I can conclude that the point is a valid operating point.

## Linearisation about selected operating point:

In order to linearize the model, I must reduce all the non-linear components to linear components, close to the operating point. In linearizing the model, offsets of the variables considered in the operating point are introduced.

Operating point,

Introducing offset variables,

From Eq(1), the equilibrium equation is

Introducing the offset variables to Eq(1)

Eliminating the derivative of the operating point

Substituting the equilibrium equation Eq(6)

Introducing the offset variables to Eq(2)

Substituting

Introducing offset variables to Eq(3)

The equilibrium equation is

Substituting into Eq(3) with the offset variables introduced and removing the steady state value of omega from the derivative.

Assuming the offset is small, it can be neglected, producing the following linearised equation

Introducing offset variables to Eq(4)

Substituting into the above equation

Assuming is small

Substituting offset variables into Eq(5)

As, and after substituting our offset variables into the above equation, as well as eliminating steady state derivatives, we arrive at the following formula.

As the offset is assumed to be small tends to . Furthermore, as is assumed to be small, tends towards the angle itself. i.e. Therefore the term becomes . Therefore the equation reduces to the following form:

## Q2 State Space Representation of System:

Using the newly found equations about our operating point that we found in Q(1) we have:

n Q(1)g duces to the following form:rating point are introduced.ose to the operating point. In linearizing the model, offsets

Now transforming the equations to the Laplace domain:

Eq(7)

Eq(8)

Eq(9)

Eq(10)

Eq(11)

By rearranging the equations 7-11 with input and output , we can determine a transfer function for the plant, we can then translate into a state space representation.

Rearranging Eq(7) in terms of

If we then substitute the expression gained above into Eq(8) we can find a new expression for in terms of **.**

Now if we rearrange the above equation to get  interms of and we have:

And substitute this into Eq(9),

If we expand the above equation,

And rearrange it in terms of,

We can finally substitute this into Eq(10) finding an expression for is found in terms of **.**

We then rearrange Eq(11) to get an equation in terms of the input and output of our plant after substituting in our expression for

By getting everything in terms of and **:**

We can now find the transfer function of the plant about the operating point:

I then substitute in the values for my operating point and get the following transfer function:

I then made the denominator monic by dividing each term by giving us the final transfer function.

We can now use the above transfer function to determine the plant’s state space representation.

Multiplying above and below by Z(s) and converting back to a differential equation

Choosing z and its’ first 3 derivatives as our state variables:

We now find the derivatives of the state variables to find the following expressions:

Which allows us to formulate the output:

Using

Therefore or matrices A, B, and C are:

I then calculated the corresponding state matrix in MATLAB.

There are two real and a complex conjugate pair of poles:

The only zero of the system is:

+ 0j

We note that as all poles lie in the left hand plane that, at the operating point, our plant is considered stable.

The operating point of the plant can be considered to be stable as all poles lie in the left hand plane.

## Controllability:

The plant’s controllability matrix has a rank of 4, i.e. it is not singular, and it has a non-zero determinant, therefore the plant is completely controllable.

## Observability:

As all the states are observable the plant is completely observable. The observability also has a full rank of n = 4.

Now the step response of our plant in series with a unit step input response is:



We note that the steady state error is 97.76%, PO% is 12.6%, and the 2% settling time is 0.445 seconds.

## Q3

I ran a bode plot of the plant. I will use this to determine to measure my closed loop gain bandwidth, and will then find T, my time constant.



The closed loop bandwidth is approximately 16 rad/sec. We can then use the rule of thumb that states:

Therefore T can be roughly estimated to be:

We note that the fastest time constant in the plant is from the pole -136.94 + j0. This is another way of determining the approximate value of T. We get its’ sampling period.

T is chosen to be as it should be less than the fastest time constant.

## Discrete-Time System:

In order to produce an equivalent discrete time system, I utilised a zero-order hold system, the transfer function of which is as follows:

We note that all the zeros lie within the unit circle and so the discrete system is minimum phase.

We also note that the DC gain of the system is quite small as before, being roughly 0.0225.

## Q4 Discrete-time linear state feedback controller with observer

We were given the following specifications for our linear state feedback controller with observer.

It has to have Zero steady state error, a 2% settling time not exceeding 0.8 seconds and a percentage overshoot of less than or equal to 20%.

To understand whether the system was stable or not I created the Pole-Zero map of the system.



As all poles lie within the unit circle we note that the system is stable.

I then checked the step response of the open-loop system to a stepped input:

It is clear to see that the system is not tracking at all, so to ensure perfect tracking I will introduce a pole at 1. The 2% settling time and percentage overshoot do meet specifications however, being 0.446 seconds and 12.6% respectively.

Now I will calculate my damping ratio, , using the percentage overshoot formula.

From the 2% settling time requirement

Using the sampling period , we can find a constraint on the z-domain location of the closed loop poles.

Therefore the modulus of the z-domain closed-loop poles must be less than 0.7788.

Transferring this discrete time system into its’ associated state space format we have:

I chose poles at 0.7, 0.6, 0.5 and 0.4 with the aim of achieving no overshoot. My damping ratio was also picked at a value of from the constraint devised earlier.

The gains of the corresponding system are determine to be:

Using the following equation:

I determined that my reference input should be scaled by 6.88.

Now finding the closed loop response of the linear state feedback controller, with the relevant poles and gains, and without observer is:



We note now that all the system specifications are met as our steady state error is now zero, our 2% settling time is 0.74 seconds and the percentage overshoot is 0%, however we have yet to add an observer into the system.

I chose poles that were at least twice as fast as the most negative pole chosen to calculate the gains K. To ensure that the estimation error from the observer converges quickly to 0 I must choose these pole locations to result in satisfactory observer gains L.

The observer target poles:

The corresponding observer gains L from determined by the placement of these poles are:

We also note that are actual poles are the same as our target poles:

Now running the step response of the closed loop system with the observer due to a stepped input we get:



We note that this is apparently the exact same response as the closed loop system without the observer. This is due to the estimation error of the observer being approximately zero. Therefore our values of L work as designed, and all states are observed with a high degree of accuracy.

## Q5 Discrete-time PID Controller:

For this controller we had the same specifications as before.

To achieve zero steady state error in the continuous time domain we must introduce a pole at 0 so using the identity, an open loop pole at 1 is required.

From Q4 the PO% and 2% settling time require that or damping ratio be, and the modulus of the z-domain closed-loop poles must be less than 0.7788.

In discrete-time a PID Controller has the following form:

We examine he root locus of the open loop system:

Now zooming in to further analyze the asymptotes:



As the relative degree of the system is 1, as there are 4 poles and 3 zeros, it is as expected that there is an asymptote at -180. As the degree of the denominator is 4 there are 4 branches of the positive root locus. When choosing the gain k we must ensure that the modulus is less than 0.7788 and within the cardioid.

I chose to place my zeroes at 0.3 and 0.4 resulting in a gain k of 29.87. As it is a digital system I can leave the gain as is as digital systems can implement precise gains.

The corresponding poles of the closed loop system are found to be:

All of these obey our design constraints.

The close system step response due to a stepped input is then:



We see that all specifications are met. We have no overshoot, the 2% settling time is 0.644 seconds and the system tracks to a steady state value of practically 1.